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INTEGRATING THE EQUATION OF GRADUALLY VARIED FLOW

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HYDRAULICS DIVISION

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INTEGRATING THE EQUATION OF GRADUALLY VARIED FLOW

Ven Te Chow,[1] A.M. ASCE

SYNOPSIS

This paper presents a new method of integrating the equation of gradually varied flow in prismatic channels. The salient features developed for the method are as follows:

1) The differential equation of gradually varied flow is integrated under given assumptions, resulting in an equation which contains varied flow functions belonging to the type same as Bakhmeteff's varied flow function.
2) Bakhmeteff's varied flow function table is expanded to triple its original size.
3) Formulas for hydraulic exponents are derived and curves of the exponents for rectangular, trapezoidal, and circular channel sections are constructed.
4) Curves expediting determination of normal and critical depths are prepared for rectangular, trapezoidal, and circular channel sections.
5) A brief survey of many existing methods of direct integration is undertaken.
6) A numerical example is given, illustrating the application of the new method.

The new method is found to be accurate, simple, and time-saving for the practical application.

INTRODUCTION

Solving the gradually varied flow equation by mathematical integration provides a direct procedure of flow computation in prismatic open channels. Many methods of direct integration have been developed for this purpose. The method presented in this paper is the outcome of a study of a number of existing methods. The author is aware that the preference of a method depends more or less upon the individual taste. Hence, the purpose of this paper is simply to introduce a new method of integration supplemented with charts and table which the author finds accurate, simple, and time-saving for the practical application.

Equation of Gradually Varied Flow

In deriving the gradually varied flow equation the following conditions are assumed:

1) The flow is steady; that is, the hydraulic characteristics of flow remain constant for the time interval under consideration.

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2) The channel is prismatic; that is, the channel has constant alignment and shape.
3) The head loss or energy slope at a section is determined by the Manning formula.
4) The slope of the channel is small so that: (a) the depth of flow is the same whether the vertical or normal (to the channel bottom) direction is used; (b) hydrostatic distribution of pressure exists throughout the depth of flow over a section; and (c) no air entrainment occurs.
5) The velocity in the direction of flow is uniformly distributed across each cross section. Hence, the velocity head in a section is represented by $V^2/2g$ in which V is the mean velocity of flow in the section.
6) The conveyance and section factor, to be defined later, are exponential functions of the depth of flow.

Figure 1 represents the flow profile for gradually varied flow in the elementary length dx of an open channel. The total head above the datum at the upstream section 1 is

$$H = z + y + \left(\frac{V^2}{2g}\right) \tag{1}$$

Differentiating this equation with respect to x, the following equation is obtained:

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \tag{2}$$

It is apparent from Fig. 1 that dH/dx is the slope of the energy grade line S and dz/dx is the slope of the channel bottom S_0. The slope is considered as positive when it declines downstream. Hence, the above equation may be written for dy/dx as

$$\frac{dy}{dx} = -\frac{S_0 - S}{1 + \frac{d}{dx}\left(\frac{V^2}{2g}\right)} \tag{3}$$

This is the general differential equation of gradually varied flow in an open channel. The term dy/dx on the left side of the equation represents the slope of water surface with respect to the channel bottom. It should be noted that the negative sign preceding the right side of the equation is not usually shown in conventional hydraulics textbooks. The negative sign which appears here simply results from the application of a rule of sign convention. As flow always involves a loss of energy, this rule dictates that the loss in head is equal to the initial value minus the final value. Hence the friction loss dH is positive as shown in the figure. By using this rule, confusion usually arising from mathematical derivation of the equation can be avoided.(1)[2]

By assumption 3, the Manning formula is applied to the gradually varied flow at section 1 of the channel in Fig. 1, with a discharge Q, depth of flow y, and slope S; or

2. Numerals in parentheses, thus: (1), refer to corresponding items in the List of References. (See Appendix I)

$$Q = K\sqrt{S} \tag{4}$$

where

$$K = \frac{1.49}{n} A R^{2/3} \tag{5}$$

in which K is known as the conveyance, A the water area, and R the hydraulic radius; all correspond to the depth y.

If a uniform flow occurs in the channel having the discharge Q, then

$$Q = K_n \sqrt{S_o} \tag{6}$$

where (7)

$$K_n = \frac{1.49}{n} A_n R_n^{2/3}$$

in which K_n is the conveyance corresponding to the normal depth of flow y_n and S_o the slope of the channel bottom.

Eliminating Q from Eqs. 4 and 6 and solving for S,

$$S = \frac{K_n^2}{K^2} S_o \tag{8}$$

If a critical flow of discharge Q occurs, the following expression, according to the theory of critical flow, may be written:

$$Q = \sqrt{g}\, Z_c \tag{9}$$

where

$$Z_c = A_c \sqrt{A_c / T_c} \tag{10}$$

in which Z_c is called the section factor, A_c the water area, and T_c the top width; all correspond to the critical depth of flow y_c. Similarly, the section factor for a depth of flow y is expressed as

$$Z = A\sqrt{A/T} \tag{11}$$

in which A and T are respectively the water area and top width corresponding to the depth y.

Since $Q = VA$ and $dA = Tdy$, the second term in the denominator of Eq. 3 may be developed as follows:

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = -\frac{Q^2}{gA^3}\frac{dA}{dy} = -\frac{Q^2 T}{g A^3} \tag{12}$$

From Eqs. 9 and 11, the above expression becomes

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{Z_c^2}{Z^2} \tag{13}$$

Substituting Eqs. 8 for S and 13 for $d(V^2/2g)/dy$ in Eq. 3, the following form for the equation of gradually varied flow is obtained:

$$\frac{dy}{dx} = -S_o \frac{1 - \dfrac{K_n^2}{K^2}}{1 - \dfrac{Z_c^2}{Z^2}} \tag{14}$$

The Hydraulic Exponents: N and M

The conveyance K and section factor Z are functions of the depth of flow y, and the squares of their values may be expressed as

$$K^2 = C_1 y^N \tag{15}$$

and

$$Z^2 = C_2 y^M \tag{16}$$

in which C_1 and C_2 are coefficients and N and M are hydraulic exponents.

Equation for N Taking logarithms of both sides of Eq. 15, or

$$2 \log_e K = \log_e C_1 + N \log_e y \tag{17}$$

and then differentiating it with respect to y, the following expression is obtained:

$$\frac{d(\log_e K)}{dy} = N \frac{d(\log_e y)}{2\, dy} = \frac{N}{2y} \tag{18}$$

Now, taking logarithms of both sides of Eq. 5 and differentiating it with respect to y, the following expression is obtained:

$$\frac{d(\log_e K)}{dy} = \frac{1}{A}\frac{dA}{dy} + \frac{2}{3}\frac{1}{R}\frac{dR}{dy} \tag{19}$$

Since $dA/dy = T$ and $R = A/P$ where P is the wetted perimeter. Eq. 19 becomes

$$\frac{d(\log_e K)}{dy} = \frac{1}{3A}\left(5T - 2R\frac{dP}{dy}\right) \tag{20}$$

Equating the right sides of Eqs. 18 and 20 and solving for N,

$$N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right) \tag{21}$$

This is the general equation for the hydraulic exponent N. For a trapezoidal channel section of bottom width b and side slopes z to 1, the values A, T, P, and R are

$$A = (b + zy)y \tag{22}$$

$$T = b + 2zy \tag{23}$$

$$P = b + 2y\sqrt{1+z^2} \tag{24}$$

$$R = \frac{(b + zy)y}{b + 2y\sqrt{1+z^2}} \tag{25}$$

Substituting these expressions in Eq. 21 and simplifying,

$$N = \frac{10}{3}\,\frac{1 + 2z(y/b)}{1 + z(y/b)} - \frac{8}{3}\,\frac{\sqrt{1+z^2}\,(y/b)}{1 + 2\sqrt{1+z^2}\,(y/b)} \tag{26}$$

This equation indicates that the value of N for the trapezoidal section is a function of z and y/b. For values of z = 0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, and 4.0, a family of curves for N vs. y/b are constructed as shown in Fig. 2.[3] These curves indicate that the value of N varies within a range of 2.0 and 5.3.

A curve for circular section with N plotted against y/D, where D is the diameter, is also shown in Fig. 2. This curve was developed by a similar procedure as mentioned above but constructed from a much more complicated formula. The curve indicates that the value of N varies within a rather narrow range for value of y/D less than 0.7 or so, but it increases rapidly as the value of y/D becomes greater than 0.7.

Equation for M Taking logarithms on both sides of Eq. 16 and then differentiating it with respect to y, the following expression is obtained,

$$\frac{d(\log_e Z)}{dy} = \frac{M}{2y} \tag{27}$$

Now, taking logarithms on both sides of Eq. 11 and differentiating it with respect to y, the following expression is obtained:

$$\frac{d(\log_e Z)}{dy} = \frac{3T}{2A} - \frac{1}{2T}\frac{dT}{dy} \tag{28}$$

Equating Eqs. 27 and 28 and solving for M,

$$M = \frac{y}{A}\left(3T - \frac{A}{T}\frac{dT}{dy}\right) \tag{29}$$

3. These curves for trapezoidal sections have been constructed by Kirpich.(2)

This is the general equation for the hydraulic exponent M. For a trapezoidal section, A and T in Eq. 29 are substituted respectively with Eqs. 22 and 23 and the resulting equation is

$$M = \frac{3[1+2z(y/b)]^2 - 2z(y/b)[1+z(y/b)]}{[1+2z(y/b)][1+z(y/b)]} \tag{30}$$

This equation is plotted for different values of z as shown in Fig. 3. These curves indicate that the value of M varies in a range from 3.0 to 5.0. The curve for circular section obtained from a more complicated formula is also shown in Fig. 3. This curve shows that the value of M decreases rapidly as the depth of flow approaches the top of the conduit.

Integration of the Gradually Varied Flow Function

From normal depth y_n and critical depth y_c, the corresponding squares of hydraulic exponents from Eqs. 15 and 16 are $K_n^2 = C_1 y_n^N$ and $Z_c^2 = C_2 y_c^M$. Substituting these equations and Eqs. 15 and 16 in Eq. 14, the gradually varied flow equation becomes

$$\frac{dy}{dx} = -S_o \frac{1-(y_n/y)^N}{1-(y_c/y)^M} \tag{31}$$

Let $u = y/y_n$, the above equation may be expressed as

$$dx = -\frac{y_n}{S_o}\left[1 - \frac{1}{1-u^N} + (y_c/y_n)^M \frac{u^{N-M}}{1-u^N}\right] du \tag{32}$$

This equation can be integrated for obtaining the length x of the flow surface curve under consideration. For a profile of gradually varied flow the change in depth within the reach under consideration is generally small. It may be assumed that the hydraulic exponents are independent of y within the range of the integrating limits. However, in cases where the hydraulics exponents are noticeably dependent on y within the limits of a given reach, the reach should be subdivided and the hydraulic exponents in each subdivided reach are assumed constant. By integrating Eq. 32, the following equation is obtained:

$$x = -\frac{y_n}{S_o}\left[u - \int_0^u \frac{du}{1-u^N} + (y_c/y_n)^M \int_0^u \frac{u^{N-M}}{1-u^N} du\right] \tag{33}$$

$$+ \text{ A CONSTANT}$$

The first integral in the right side of the above equation is designated by F(u,N), or

$$F(u,N) = \int_0^u \frac{du}{1-u^N} \tag{34}$$

which is known as the varied flow function.

The second integral in Eq. 33 may also be expressed in the same form of the preceding varied flow function. Let $v = u^J$ and $J = N/(N-M+1)$, this integral can be transformed as

$$\int_0^u \frac{u^{N-M}}{1-u^N}\,du = \frac{J}{N}\int_0^v \frac{dv}{1-v^J} = \frac{J}{N}F(v,J) \tag{35}$$

in which

$$F(v,J) = \int_0^v \frac{dv}{1-v^J} \tag{36}$$

This is a varied flow function same as $F(u,N)$ except that the variables u and N are replaced by v and J respectively.

Using the notations for varied flow functions, Eq. 33 may be written as

$$X = -\frac{y_n}{S_0}\left[u - F(u,N) + (y_c/y_n)^M (J/N) F(v,J)\right] \tag{37}$$

$$+ \text{ A CONSTANT}$$

in which $u = y/y_n$, $v = u^J$, $J = N/(N-M+1)$ and $F(u,N)$ and $F(v,J)$ are varied flow functions.

By Eq. 37, the length of water surface curve between two sections 1 and 2 is equal to

$$\begin{aligned} L &= x_2 - x_1 \\ &= \frac{y_n}{S_0}\Big\{(u_1 - u_2) - \left[F(u_1,N) - F(u_2,N)\right] \\ &\qquad + (y_c/y_n)^M (J/N)\left[F(v_1,J) - F(v_2,J)\right]\Big\} \end{aligned} \tag{38}$$

in which the subscripts 1 and 2 refer to sections 1 and 2 correspondingly.

Existing Methods of Integration

The differential equation of gradually varied flow cannot be expressed explicitly in terms of y for all types of channel cross section, and hence, a direct and exact integration of the equation is practically impossible. Many attempts have been made either to solve the equation for few special cases or to introduce certain assumptions, making the equation amenable to a mathematical integration. Table 1 is a list of many existing methods of direct integration, arranged chronologically. Although the list is incomplete, it provides a general idea on the development of direct integration method for the solution of the gradually varied flow equation. It is noted that most of the early methods were developed for channels of a specific channel cross section;

while recent solutions, since Bakhmeteff, were designed for channels of all shapes. Most early methods use Chezy's formula, whereas later methods use Manning's formula.

In the Bakhmeteff method(13) the channel length under consideration is divided into short reaches. The change of kinetic energy in each short reach is assumed constant, and the integration is carried out by steps of short range. With an attempt to improve Bakhmeteff's method, Mononobe(15) introduced two assumptions for hydraulic exponents. By these assumptions the effects of changing velocity, friction head, and channel shape are taken into account integrally without any necessity of dividing the channel length into short reaches. Thus, the Mononobe method affords a more direct and accurate computation procedure whereby results can be obtained without recourse to successive steps. In applying this method to practical problems, however, it will be found that the first assumption is not very satisfactory; hence, an error in the results may be introduced by assuming the wetted perimeter to be a monomial function of the depth. The chief drawback in this method perhaps lies in the difficulty in using the charts which are not accurate enough for practical purpose.

Lee(16)(17) and Von Seggern(18) suggested new assumptions which result in more satisfactory solutions. The Von Seggern method introduces a new varied flow function in addition to the function used by Bakhmeteff, and hence, an additional table for the new function is required. The Lee method, however, is similar to the present method, involving varied flow functions of the Bakhmeteff type.

All the above methods assume constant hydraulic exponents. This assumption is satisfactory to obtain approximate solutions for flow in most open conduits. The closed conduit presents a different situation in which the hydraulic exponents vary rapidly as the water depth approaches the crown, (for example, see Figs. 2 and 3 for circular conduit) thus rendering the basic assumption inapplicable. For circular conduits, Keifer and Chu(19) proposed a method of exact integration. For other shapes of closed channel sections, this method may be used, but different tables and charts are required.

The Table of Varied Flow Function

Solution of Eq. 38 is simplified by the use of a table of the varied flow function. The preparation of this table was undertaken and performed for the first time during 1914 to 1915 by the Research Board of the then Russian Reclamation Service under the direction of Boris A. Bakhmeteff, then Professor of General and Advanced Hydraulics at Polytechnic Institute Emperor Peter the Great, St. Petersburg, Russia. It was told that the work involved a long and tedious procedure.(13)[4] In the turmoil of the Russian Revolution in 1917, the table so computed became unavailable, so the task of computing was done over again by Professor Kholodovsky and partly by Dr. Pestrečov. The recomputed table was more precise and complete, covering a range of N from 2.8 to 5.4. This table was published in 1932(13) when the author, Bakhmeteff, was Professor of Civil Engineering at the Columbia University.

The value of the variable J in Eq. 36 usually covers quite a broad range. The varied flow function table is therefore expanded to cover a range of N

4. Methods of computing the varied flow function table are explained in Bakhmeteff's "Hydraulics of Open Channels" (13) pp. 303-305. A negative sign should be placed in front of the right side of the equation as printed on the bottom of p. 305.

from 2.2 to 9.8, almost triple the original size. This expanded table[5] is included in the present paper as Table 2. It should be noted, however, that the accuracy of the present method will depend on determining accurate values of the varied flow function from this table. In order to attain a higher degree of accuracy, the figures presented in the table should be carried to four or five decimal places and the interval of N should be reduced. It is believed that this refinement can be achieved with the aid of an electronic digital computor, whereas the manual computation will become undoubtedly impracticable.

Application of the Method

The surface profile of a gradually varied flow can be determined by the method presented herewith. The channel under consideration is first divided into a number of reaches. Then, the length of each reach is computed by Eq. 38 from known or assumed depths. The procedure of computation is as follows:

1) Compute the normal depth y_n and critical depth y_c from the given data Q and S_o. For rectangular, trapezoidal, and circular channels, these depths may be determined by means of Figs. 4 and 5.

Compute $K_n = Q/\sqrt{S_o}$ from Eq. 6, $A_n R_n^{2/3} = n\, K_n/1.49$ from Eq. 7, and then $A_n R_n^{2/3} / b^{8/3}$. From Fig. 4, find the corresponding ratio y/b or y/D. Multiplying this ratio by b or D, the normal depth is obtained.

Compute $Z_c = Q/\sqrt{g}$ from Eq. 9, and the value of $Z_c/b^{2.5}$. From Fig. 3, find the corresponding ratio y/b or y/D. Multiplying this ratio by b or D the original depth is obtained.

2) Determine the hydraulic exponents N and M for an estimated average depth of flow in the reach under consideration.

For rectangular, trapezoidal, and circular channels, values of N and M may be obtained directly from the curves of Figs. 2 and 3. It should be noted that values of hydraulic exponents vary rapidly as the depth of flow approaches the crown of the closed conduit, such as a circular conduit. In this situation, short reaches should be taken in the computation.

For channel sections other than those given in Figs. 2 and 3, the values of N and M may be determined by Eqs. 21 and 29. Approximate values, however, may be obtained from the following formulas:

$$N = 2\,\frac{\log_{10}(K/K_n)}{\log_{10}(y/y_n)} \tag{39}$$

and

$$M = 2\,\frac{\log_{10}(Z/Z_c)}{\log_{10}(y/y_c)} \tag{40}$$

in which K and Z correspond to the average depth y or other depth of flow in the channel. These formulas can be easily derived from Eqs. 15 and 16.

5. Values in Bakhmeteff's table were checked and several errors were found. This table, covering the range of N from 2.8 to 5.4, is reproduced in Table 2 with the errors corrected. The permission for reproduction of this portion of table values was granted by the publisher, McGraw-Hill Book Company, Inc. of New York on December 3, 1954.

The hydraulic exponents may also be determined by a graphical method. This involves a logarithmic plotting of K and Z as ordinates against the depth y as abscissa. The hydraulic exponent is equal to twice the slope of the plotted straight line. For depth approaching the crown of a closed conduit, the plot appears as a curve, and the hydraulic exponent is equal to twice the slope of the tangent to the curve at the given depth.

3) Compute J by $J = N/(N-M+1)$.

4) Compute values of $u = y/y_n$ and $v = u^J$ at two end sections of the reach.

5) From the varied flow function table, Table 2, find values of $F(u,N)$ and $F(v,J)$.

6) Compute the length of the reach by Eq. 38.

The above procedure is illustrated by the following example:

Example In a trapezoidal channel, having $Q = 400$ cfs., $b = 20$ ft., $z = 2$, $S_o = 0.0016$, and $n = 0.025$, a depth of flow of 5 ft. occurs just upstream of a dam. Determine the length of the backwater curve extending from the dam site to an upstream section where the depth of flow is 1 per cent greater than the normal depth.

Solution 1) Substituting the given data in Eq. 6, $K_n = 400 / \sqrt{0.0016} = 10{,}000$. By Eq. 7, $A_nR_n^{2/3} = (0.025 \times 10{,}000)/1.49 = 167.7$. The value of $A_nR_n^{2/3} / b^{8/3} = 0.0569$. From Fig. 4, it is found that $y_n/b = 0.168$ or $y_n = 3.36$ ft.

From Eq. 9, $Z_c = 400/\sqrt{g} = 70.5$. The value of $Z_c/b^{2.5} = 0.0394$. From Fig. 5, it is found that $y_c/b = 0.108$ or $y_c = 2.16$ ft.

2) The depth at the downstream end of the backwater curve is $y_1 = 5$ ft. At the upstream end, it is $y_2 = 1.01 \times 3.36 = 3.40$ ft. The average depth may be taken as 4.20 ft. and $y/b = 0.21$. From Figs. 2 and 3, the hydraulic exponents are found to be $N = 3.65$ abd $M = 3.43$.

3) The value of $J = 3.65/(3.65-3.43+1) = 2.99$.

4) For each section, values of u and v are computed as given in the second and third columns of the following table:

y	u	v	F(u,N)	F(v,J)
5.00	1.488	3.280	0.148	0.049
3.40	1.012	1.036	1.025	1.010
Diff.:	0.476		-0.877	-0.961

5) The varied flow functions $F(u,N)$ and $F(v,J)$ are obtained from Table 2 and given in the fourth and fifth columns of the above table.

6) In Eq. 38, $y_n/S_o = 2{,}100$ and $(y_c/y_n)^M(J/N) = 0.180$. The length of the backwater curve is therefore

$$L = 2{,}100 \left[0.476-(-0.877)+(0.180)(-0.961)\right] = 2{,}480 \text{ ft.}$$

CONCLUSIONS

The method presented in this paper has the following features:

1) The effect of change in kinetic energy is considered.

2) The method is applicable to channels of any shape. For depth of flow approaching the crown of a closed conduit, the hydraulic exponents can be obtained from curves, and short reaches should be taken in the computation for desired accuracy.

3) The resulting equation of integration contains single-type flow functions. Hence, only a single table is necessary for their evaluation.

4) The varied flow function table covers a sufficiently wide range for extended applications.

5) By the use of curves, determination of hydraulic exponents and the normal and critical depths are simplified for rectangular, trapezoidal, and circular channel sections which are most frequently encountered. Curves could be prepared as needed for other channel shapes.

6) Because only a single-type varied flow function is involved, and the procedure is expedited by the use of charts and table,[6] the new method is found to be simple and time-saving.

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APPENDIX II. NOMENCLATURE

The following letter symbols are adopted for use in the paper:

A	the water area of a cross section, in sq. ft.
A_c	the water area at critical depth, in sq. ft.
A_n	the water area at normal depth, in sq. ft.
b	bottom width of a trapezoidal section, in ft.
C, C_1, C_2	numerical coefficients of a formula.
D	diameter of a circular conduit, in ft.
e	the base of natural logarithms.
$F(u,N)$	$\int_0^u \frac{du}{1 - u^N}$, a varied flow function.
$F(v,J)$	$\int_0^v \frac{dv}{1 - v^J}$, a varied flow function.
g	gravitational acceleration.
H	the total head of flow in a cross section above a datum, in ft.
J	$N/(N-M+1)$, a variable in the varied flow function $F(v,J)$.
K	$\frac{1.49}{n} AR^{2/3}$, the conveyance of a cross section.
K_n	the conveyance at normal depth.
L	length of a channel reach, in ft.

M	the hydraulic exponent for section factor.
m	numerical exponent in a velocity formula.
N	the hydraulic exponent for conveyance.
n	Manning's coefficient of roughness.
P	the wetted perimeter of a cross section, in ft.
p	numerical exponent in a velocity formula.
Q	discharge in cfs.
R	the hydraulic radius of a cross section, in ft.
R_n	the hydraulic radius at normal depth, in ft.
S	Slope of the energy grade line.
S_o	longitudinal slope of the channel bottom.
T	the top width of the water area, in ft.
T_c	the top width at critical depth, in ft.
u	y/y_n
V	average velocity of flow in a section, in fps.
v	u^J, a variable in the varied flow function $F(v,J)$.
x	distance along the channel bottom from an arbitrary origin, in ft., measured either downstream or upstream.
y	depth of flow, in ft.
y_c	critical depth of flow, in ft.
Z	$A \sqrt{A/T}$, the section factor of a cross section.
Z_c	the section factor at critical depth.
z	elevation of the channel bottom above a datum, in ft.; or the horizontal projection of the side slope of a trapezoidal channel when the vertical projection is unity; or a variable in Baticle's method.

Table 1 - Existing Methods of Integrating Gradually Varied Flow Equation

Year of Publication	Investigator	Type of Channel	Effect of Change in Kinetic Energy	Velocity Formula	Assumptions for Hydraulic Exponents	Reference
1848	Dupuit	Broad rectangle	Ignored	Chezy	N = 3, M = 3	(3)
1860	Bresse	Broad rectangle	Considered	Chezy	N = 3, M = 3	(4)
1875	Grashof	Broad rectangle	Considered	Chezy	N = 3, M = 3	(5)
1880	Rühlmann	Broad rectangle	Ignored	Chezy	N = 3, M = 3	(6)
1898	Tolkmitt	Broad parabola	Considered	Chezy	N = 4, M = 3.5	(7)
1900	Masoni	Common rectangle	Considered	Chezy	N = 3, M = 3	(8)
1914	Schaffernak Ehrenberger	Broad rectangle	Ignored	$C\,y^{0.75}\,S^{0.5}$ $23.78\,y^{0.776}\,S^{0.458}$ $22.11\,y^{0.58}\,S^{0.43}$	N = 3.5, M = 3 N = 3.552, M = 3 N = 3.16, M = 3	(9)
1921	Baticle	Approximate trapezoid	Ignored	Chezy	$K^2 = z^5 = A^2R$ where z is a variable and dy/dz = constant	(10)

Table 1 - Existing Methods of Integrating Gradually Varied Flow Equation (Continued)

Year of Publication	Investigator	Type of Channel	Effect of Change in Kinetic Energy	Velocity Formula	Assumptions for Hydraulic Exponents	Reference
1928	Kozény	Broad rectangle	Considered	$C\,y^{0.7}\,S^{0.5}$	$N = 3.4$, $M = 3$	(11)
1930	Schoklisch	Broad rectangle	Ignored	$C\,R^{m}\,S^{p}$ where m and p are exponents	$N = 2+2m$, $M = 3$	(12)
1932 (1912)	Bakhmeteff	All shapes	Considered by steps	Chezy	$K^2 \propto y^N$	(13)(14)
1938	Mononobe	All shapes	Considered	$C\,R^{m}\,S^{0.5}$ where m is an exponent	$P \propto y^{const.}$ $A^2 \propto y^{const.}$	(15)
1947	Lee	All shapes	Considered	Manning	$K^2 \propto y^N$ $A^2 \propto y^{const.}$	(16)(17)
1950	Von Seggern	All shapes	Considered	Manning	$K^2 \propto y^N$ $Z^2 \propto y^M$	(18)
1954	Keifer-Chu	Circular, but extensible to other shapes	Considered	Manning	None	(19)

Table 2 - The Varied Flow Function Table (p.1 of 12)

u \ N	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.02	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
0.04	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.06	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060
0.08	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080
0.10	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
0.12	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120
0.14	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140
0.16	0.161	0.161	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160
0.18	0.181	0.181	0.181	0.180	0.180	0.180	0.180	0.180	0.180	0.180
0.20	0.202	0.201	0.201	0.201	0.200	0.200	0.200	0.200	0.200	0.200
0.22	0.223	0.222	0.221	0.221	0.221	0.220	0.220	0.220	0.220	0.220
0.24	0.244	0.243	0.242	0.241	0.241	0.241	0.240	0.240	0.240	0.240
0.26	0.265	0.263	0.262	0.262	0.261	0.261	0.261	0.260	0.260	0.260
0.28	0.286	0.284	0.283	0.282	0.282	0.281	0.281	0.281	0.280	0.280
0.30	0.307	0.305	0.304	0.303	0.302	0.302	0.301	0.301	0.301	0.300
0.32	0.329	0.326	0.325	0.324	0.323	0.322	0.322	0.321	0.321	0.321
0.34	0.351	0.348	0.346	0.344	0.343	0.343	0.342	0.342	0.341	0.341
0.36	0.372	0.369	0.367	0.366	0.364	0.363	0.363	0.362	0.362	0.361
0.38	0.395	0.392	0.389	0.387	0.385	0.384	0.383	0.383	0.382	0.382
0.40	0.418	0.414	0.411	0.408	0.407	0.405	0.404	0.403	0.403	0.402
0.42	0.442	0.437	0.433	0.430	0.428	0.426	0.425	0.424	0.423	0.423
0.44	0.465	0.460	0.456	0.452	0.450	0.448	0.446	0.445	0.444	0.443
0.46	0.489	0.483	0.479	0.475	0.472	0.470	0.468	0.466	0.465	0.464
0.48	0.514	0.507	0.502	0.497	0.494	0.492	0.489	0.488	0.486	0.485
0.50	0.539	0.531	0.525	0.521	0.517	0.514	0.511	0.509	0.508	0.506
0.52	0.565	0.557	0.550	0.544	0.540	0.536	0.534	0.531	0.529	0.528
0.54	0.592	0.582	0.574	0.568	0.563	0.559	0.556	0.554	0.551	0.550
0.56	0.619	0.608	0.599	0.593	0.587	0.583	0.579	0.576	0.574	0.572
0.58	0.648	0.635	0.626	0.618	0.612	0.607	0.603	0.599	0.596	0.594
0.60	0.676	0.663	0.653	0.644	0.637	0.631	0.627	0.623	0.620	0.617
0.61	0.691	0.678	0.667	0.657	0.650	0.644	0.639	0.635	0.631	0.628
0.62	0.706	0.692	0.680	0.671	0.663	0.657	0.651	0.647	0.643	0.640
0.63	0.722	0.707	0.694	0.684	0.676	0.669	0.664	0.659	0.655	0.652
0.64	0.738	0.722	0.709	0.698	0.690	0.683	0.677	0.672	0.667	0.664
0.65	0.754	0.737	0.724	0.712	0.703	0.696	0.689	0.684	0.680	0.676
0.66	0.771	0.753	0.738	0.727	0.717	0.709	0.703	0.697	0.692	0.688
0.67	0.787	0.769	0.754	0.742	0.731	0.723	0.716	0.710	0.705	0.701
0.68	0.804	0.785	0.769	0.757	0.746	0.737	0.729	0.723	0.718	0.713
0.69	0.822	0.804	0.785	0.772	0.761	0.751	0.743	0.737	0.731	0.726

Table 2 - The Varied Flow Function Table (p.2 of 12)

u \ N	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
0.70	0.840	0.819	0.802	0.787	0.776	0.766	0.757	0.750	0.744	0.739
0.71	0.858	0.836	0.819	0.804	0.791	0.781	0.772	0.764	0.758	0.752
0.72	0.878	0.855	0.836	0.820	0.807	0.796	0.786	0.779	0.772	0.766
0.73	0.898	0.874	0.854	0.837	0.823	0.811	0.802	0.793	0.786	0.780
0.74	0.918	0.892	0.868	0.854	0.840	0.827	0.817	0.808	0.800	0.794
0.75	0.940	0.913	0.890	0.872	0.857	0.844	0.833	0.823	0.815	0.808
0.76	0.961	0.933	0.909	0.890	0.874	0.861	0.849	0.839	0.830	0.823
0.77	0.985	0.954	0.930	0.909	0.892	0.878	0.866	0.855	0.846	0.838
0.78	1.007	0.976	0.950	0.929	0.911	0.896	0.883	0.872	0.862	0.854
0.79	1.031	0.998	0.971	0.949	0.930	0.914	0.901	0.889	0.879	0.870
0.80	1.056	1.022	0.994	0.970	0.950	0.934	0.919	0.907	0.896	0.887
0.81	1.083	1.046	1.017	0.992	0.971	0.954	0.938	0.925	0.914	0.904
0.82	1.110	1.072	1.041	1.015	0.993	0.974	0.958	0.945	0.932	0.922
0.83	1.139	1.099	1.067	1.039	1.016	0.996	0.979	0.965	0.952	0.940
0.84	1.171	1.129	1.094	1.064	1.040	1.019	1.001	0.985	0.972	0.960
0.85	1.201	1.157	1.121	1.091	1.065	1.043	1.024	1.007	0.993	0.980
0.86	1.238	1.192	1.153	1.119	1.092	1.068	1.048	1.031	1.015	1.002
0.87	1.272	1.223	1.182	1.149	1.120	1.095	1.074	1.055	1.039	1.025
0.88	1.314	1.262	1.228	1.181	1.151	1.124	1.101	1.081	1.064	1.049
0.89	1.357	1.302	1.255	1.216	1.183	1.155	1.131	1.110	1.091	1.075
0.90	1.401	1.343	1.294	1.253	1.218	1.189	1.163	1.140	1.120	1.103
0.91	1.452	1.389	1.338	1.294	1.257	1.225	1.197	1.173	1.152	1.133
0.92	1.505	1.438	1.351	1.340	1.300	1.266	1.236	1.210	1.187	1.166
0.93	1.564	1.493	1.435	1.391	1.348	1.311	1.279	1.251	1.226	1.204
0.94	1.645	1.568	1.504	1.449	1.403	1.363	1.328	1.297	1.270	1.246
0.950	1.737	1.652	1.582	1.518	1.467	1.423	1.385	1.352	1.322	1.296
0.960	1.833	1.741	1.665	1.601	1.545	1.497	1.454	1.417	1.385	1.355
0.970	1.969	1.866	1.780	1.707	1.644	1.590	1.543	1.501	1.464	1.431
0.975	2.055	1.945	1.853	1.773	1.707	1.649	1.598	1.554	1.514	1.479
0.980	2.164	2.045	1.946	1.855	1.783	1.720	1.666	1.617	1.575	1.536
0.985	2.294	2.165	2.056	1.959	1.880	1.812	1.752	1.699	1.652	1.610
0.990	2.477	2.333	2.212	2.106	2.017	1.940	1.873	1.814	1.761	1.714
0.995	2.792	2.621	2.478	2.355	2.250	2.159	2.079	2.008	1.945	1.889
0.999	3.523	3.292	3.097	2.931	2.788	2.663	2.554	2.457	2.370	2.293
1.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Table 2 - The Varied Flow Function Table (p.3 of 12)

u \ N	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
1.001	3.317	2.931	2.640	2.399	2.184	2.008	1.856	1.725	1.610	1.508
1.005	2.587	2.266	2.022	1.818	1.649	1.506	1.384	1.279	1.188	1.107
1.010	2.273	1.977	1.757	1.572	1.419	1.291	1.182	1.089	1.007	0.936
1.015	2.090	1.807	1.602	1.428	1.286	1.166	1.065	0.978	0.902	0.836
1.020	1.961	1.711	1.493	1.327	1.191	1.078	0.982	0.900	0.828	0.766
1.03	1.779	1.531	1.340	1.186	1.060	0.955	0.866	0.790	0.725	0.668
1.04	1.651	1.410	1.232	1.086	0.967	0.868	0.785	0.714	0.653	0.600
1.05	1.552	1.334	1.150	1.010	0.896	0.802	0.723	0.656	0.598	0.548
1.06	1.472	1.250	1.082	0.948	0.838	0.748	0.672	0.608	0.553	0.506
1.07	1.404	1.195	1.026	0.896	0.790	0.703	0.630	0.569	0.516	0.471
1.08	1.346	1.139	0.978	0.851	0.749	0.665	0.595	0.535	0.485	0.441
1.09	1.295	1.089	0.935	0.812	0.713	0.631	0.563	0.506	0.457	0.415
1.10	1.250	1.050	0.897	0.777	0.681	0.601	0.536	0.480	0.433	0.392
1.11	1.209	1.014	0.864	0.746	0.652	0.575	0.511	0.457	0.411	0.372
1.12	1.172	0.981	0.833	0.718	0.626	0.551	0.488	0.436	0.392	0.354
1.13	1.138	0.950	0.805	0.692	0.602	0.529	0.468	0.417	0.374	0.337
1.14	1.107	0.921	0.780	0.669	0.581	0.509	0.450	0.400	0.358	0.322
1.15	1.078	0.892	0.756	0.647	0.561	0.490	0.432	0.384	0.343	0.308
1.16	1.052	0.870	0.734	0.627	0.542	0.473	0.417	0.369	0.329	0.295
1.17	1.027	0.850	0.713	0.608	0.525	0.458	0.402	0.356	0.317	0.283
1.18	1.003	0.825	0.694	0.591	0.509	0.443	0.388	0.343	0.305	0.272
1.19	0.981	0.810	0.676	0.574	0.494	0.429	0.375	0.331	0.294	0.262
1.20	0.960	0.787	0.659	0.559	0.480	0.416	0.363	0.320	0.283	0.252
1.22	0.922	0.755	0.628	0.531	0.454	0.392	0.341	0.299	0.264	0.235
1.24	0.887	0.725	0.600	0.505	0.431	0.371	0.322	0.281	0.248	0.219
1.26	0.855	0.692	0.574	0.482	0.410	0.351	0.304	0.265	0.233	0.205
1.28	0.827	0.666	0.551	0.461	0.391	0.334	0.288	0.250	0.219	0 193
1.30	0.800	0.644	0.530	0.442	0.373	0.318	0.274	0.237	0.207	0.181
1.32	0.775	0.625	0.510	0.424	0.357	0.304	0.260	0.225	0.196	0.171
1.34	0.752	0.605	0.492	0.408	0.342	0.290	0.248	0.214	0.185	0.162
1.36	0.731	0.588	0.475	0.393	0.329	0.278	0.237	0.204	0.176	0.153
1.38	0.711	0.567	0.459	0.378	0.316	0.266	0.226	0.194	0.167	0.145
1.40	0.692	0.548	0.444	0.365	0.304	0.256	0.217	0.185	0.159	0.138
1.42	0.674	0.533	0.431	0.353	0.293	0.246	0.208	0.177	0.152	0.131
1.44	0.658	0.517	0.417	0.341	0.282	0.236	0.199	0.169	0.145	0.125
1.46	0.642	0.505	0.405	0.330	0.273	0.227	0.191	0.162	0.139	0.119
1.48	0.627	0.493	0.394	0.320	0.263	0.219	0.184	0.156	0.133	0.113
1.50	0.613	0.480	0.383	0.310	0.255	0.211	0.177	0.149	0.127	0.108
1.55	0.580	0.451	0.358	0.288	0.235	0.194	0.161	0.135	0.114	0.097
1.60	0.551	0.425	0.335	0.269	0.218	0.179	0.148	0.123	0.103	0.087

Table 2 - The Varied Flow Function Table (p.4 of 12)

u \ N	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
1.65	0.525	0.402	0.316	0.251	0.203	0.165	0.136	0.113	0.094	0.079
1.70	0.501	0.381	0.298	0.236	0.189	0.153	0.125	0.103	0.086	0.072
1.75	0.480	0.362	0.282	0.222	0.177	0.143	0.116	0.095	0.079	0.065
1.80	0.460	0.349	0.267	0.209	0.166	0.133	0.108	0.088	0.072	0.060
1.85	0.442	0.332	0.254	0.198	0.156	0.125	0.100	0.082	0.067	0.055
1.90	0.425	0.315	0.242	0.188	0.147	0.117	0.094	0.076	0.062	0.050
1.95	0.409	0.304	0.231	0.178	0.139	0.110	0.088	0.070	0.057	0.046
2.00	0.395	0.292	0.221	0.169	0.132	0.104	0.082	0.066	0.053	0.043
2.10	0.369	0.273	0.202	0.154	0.119	0.092	0.073	0.058	0.046	0.037
2.20	0.346	0.253	0.186	0.141	0.107	0.083	0.065	0.051	0.040	0.032
2.3	0.326	0.235	0.173	0.129	0.098	0.075	0.058	0.045	0.035	0.028
2.4	0.308	0.220	0.160	0.119	0.089	0.068	0.052	0.040	0.031	0.024
2.5	0.292	0.207	0.150	0.110	0.082	0.062	0.047	0.036	0.028	0.022
2.6	0.277	0.197	0.140	0.102	0.076	0.057	0.043	0.033	0.025	0.019
2.7	0.264	0.188	0.131	0.095	0.070	0.052	0.039	0.029	0.022	0.017
2.8	0.252	0.176	0.124	0.089	0.065	0.048	0.036	0.027	0.020	0.015
2.9	0.241	0.166	0.117	0.083	0.060	0.044	0.033	0.024	0.018	0.014
3.0	0.230	0.159	0.110	0.078	0.056	0.041	0.030	0.022	0.017	0.012
3.5	0.190	0.126	0.085	0.059	0.041	0.029	0.021	0.015	0.011	0.008
4.0	0.161	0.104	0.069	0.046	0.031	0.022	0.015	0.010	0.007	0.005
4.5	0.139	0.087	0.057	0.037	0.025	0.017	0.011	0.008	0.005	0.004
5.0	0.122	0.076	0.048	0.031	0.020	0.013	0.009	0.006	0.004	0.003
6.0	0.098	0.060	0.036	0.022	0.014	0.009	0.006	0.004	0.002	0.002
7.0	0.081	0.048	0.028	0.017	0.010	0.006	0.004	0.002	0.002	0.001
8.0	0.069	0.040	0.022	0.013	0.008	0.005	0.003	0.002	0.001	0.001
9.0	0.060	0.034	0.019	0.011	0.006	0.004	0.002	0.001	0.001	0.000
10.0	0.053	0.028	0.016	0.009	0.005	0.003	0.002	0.001	0.001	0.000
20.0	0.023	0.018	0.011	0.006	0.002	0.001	0.001	0.000	0.000	0.000

Table 2 - The Varied Flow Function Table (p.5 of 12)

n \ N	4.2	4.6	5.0	5.4	5.8	6.2	6.6	7.0	7.4	7.8
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.02	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
0.04	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.06	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060
0.08	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080
0.10	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
0.12	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120
0.14	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140
0.16	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160
0.18	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180
0.20	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
0.22	0.220	0.220	0.220	0.220	0.220	0.220	0.220	0.220	0.220	0.220
0.24	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240
0.26	0.260	0.260	0.260	0.260	0.260	0.260	0.260	0.260	0.260	0.260
0.28	0.280	0.280	0.280	0.280	0.280	0.280	0.280	0.280	0.280	0.280
0.30	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300
0.32	0.321	0.320	0.320	0.320	0.320	0.320	0.320	0.320	0.320	0.320
0.34	0.341	0.340	0.340	0.340	0.340	0.340	0.340	0.340	0.340	0.340
0.36	0.361	0.361	0.360	0.360	0.360	0.360	0.360	0.360	0.360	0.360
0.38	0.381	0.381	0.381	0.380	0.380	0.380	0.380	0.380	0.380	0.380
0.40	0.402	0.401	0.401	0.400	0.400	0.400	0.400	0.400	0.400	0.400
0.42	0.422	0.421	0.421	0.421	0.420	0.420	0.420	0.420	0.420	0.420
0.44	0.443	0.442	0.441	0.441	0.441	0.441	0.440	0.440	0.440	0.440
0.46	0.463	0.462	0.462	0.461	0.461	0.461	0.460	0.460	0.460	0.460
0.48	0.484	0.483	0.482	0.481	0.481	0.481	0.480	0.480	0.480	0.480
0.50	0.505	0.504	0.503	0.502	0.501	0.501	0.501	0.500	0.500	0.500
0.52	0.527	0.525	0.523	0.522	0.522	0.521	0.521	0.521	0.520	0.520
0.54	0.548	0.546	0.544	0.543	0.542	0.542	0.541	0.541	0.541	0.541
0.56	0.570	0.567	0.565	0.564	0.563	0.562	0.562	0.561	0.561	0.561
0.58	0.592	0.589	0.587	0.585	0.583	0.583	0.582	0.582	0.581	0.581
0.60	0.614	0.611	0.608	0.606	0.605	0.604	0.603	0.602	0.602	0.601
0.61	0.626	0.622	0.619	0.617	0.615	0.614	0.613	0.612	0.612	0.611
0.62	0.637	0.633	0.630	0.628	0.626	0.625	0.624	0.623	0.622	0.622
0.63	0.649	0.644	0.641	0.638	0.636	0.635	0.634	0.633	0.632	0.632
0.64	0.661	0.656	0.652	0.649	0.647	0.646	0.645	0.644	0.643	0.642
0.65	0.673	0.667	0.663	0.660	0.658	0.656	0.655	0.654	0.653	0.653
0.66	0.685	0.679	0.675	0.672	0.669	0.667	0.666	0.665	0.664	0.663
0.67	0.697	0.691	0.686	0.683	0.680	0.678	0.676	0.675	0.674	0.673
0.68	0.709	0.703	0.698	0.694	0.691	0.689	0.687	0.686	0.685	0.684
0.69	0.722	0.715	0.710	0.706	0.703	0.700	0.698	0.696	0.695	0.694

Table 2 - The Varied Flow Function Table (p.6 of 12)

u \ N	4.2	4.6	5.0	5.4	5.8	6.2	6.6	7.0	7.4	7.8
0.70	0.735	0.727	0.722	0.717	0.714	0.712	0.710	0.708	0.706	0.705
0.71	0.748	0.740	0.734	0.729	0.726	0.723	0.721	0.719	0.717	0.716
0.72	0.761	0.752	0.746	0.741	0.737	0.734	0.732	0.730	0.728	0.727
0.73	0.774	0.765	0.759	0.753	0.749	0.746	0.743	0.741	0.739	0.737
0.74	0.788	0.779	0.771	0.766	0.761	0.757	0.754	0.752	0.750	0.748
0.75	0.802	0.792	0.784	0.778	0.773	0.769	0.766	0.763	0.761	0.759
0.76	0.817	0.806	0.798	0.791	0.786	0.782	0.778	0.775	0.773	0.771
0.77	0.831	0.820	0.811	0.804	0.798	0.794	0.790	0.787	0.784	0.782
0.78	0.847	0.834	0.825	0.817	0.811	0.806	0.802	0.799	0.796	0.794
0.79	0.862	0.849	0.839	0.831	0.824	0.819	0.815	0.811	0.808	0.805
0.80	0.878	0.865	0.854	0.845	0.838	0.832	0.828	0.823	0.820	0.818
0.81	0.895	0.881	0.869	0.860	0.852	0.846	0.841	0.836	0.833	0.830
0.82	0.913	0.897	0.885	0.875	0.866	0.860	0.854	0.850	0.846	0.842
0.83	0.931	0.914	0.901	0.890	0.881	0.874	0.868	0.863	0.859	0.855
0.84	0.949	0.932	0.918	0.906	0.897	0.889	0.882	0.877	0.872	0.868
0.85	0.969	0.950	0.935	0.923	0.912	0.905	0.898	0.891	0.887	0.882
0.86	0.990	0.970	0.954	0.940	0.930	0.921	0.913	0.906	0.901	0.896
0.87	1.012	0.990	0.973	0.959	0.947	0.937	0.929	0.922	0.916	0.911
0.88	1.035	1.012	0.994	0.978	0.966	0.955	0.946	0.938	0.932	0.927
0.89	1.060	1.035	1.015	0.999	0.986	0.974	0.964	0.956	0.949	0.943
0.90	1.087	1.060	1.039	1.021	1.007	0.994	0.984	0.974	0.967	0.960
0.91	1.116	1.088	1.064	1.045	1.029	1.016	1.003	0.995	0.986	0.979
0.92	1.148	1.117	1.092	1.072	1.054	1.039	1.027	1.016	1.006	0.999
0.93	1.184	1.151	1.123	1.101	1.081	1.065	1.050	1.040	1.029	1.021
0.94	1.225	1.188	1.158	1.134	1.113	1.095	1.080	1.066	1.054	1.044
0.950	1.272	1.232	1.199	1.172	1.148	1.128	1.111	1.097	1.084	1.073
0.960	1.329	1.285	1.248	1.217	1.188	1.167	1.149	1.133	1.119	1.106
0.970	1.402	1.351	1.310	1.275	1.246	1.319	1.197	1.179	1.162	1.148
0.975	1.447	1.393	1.348	1.311	1.280	1.250	1.227	1.207	1.190	1.173
0.980	1.502	1.443	1.395	1.354	1.339	1.288	1.262	1.241	1.221	1.204
0.985	1.573	1.508	1.454	1.409	1.372	1.337	1.309	1.284	1.263	1.243
0.990	1.671	1.598	1.537	1.487	1.444	1.404	1.373	1.344	1.319	1.297
0.995	1.838	1.751	1.678	1.617	1.565	1.519	1.479	1.451	1.416	1.388
0.999	2.223	2.101	2.002	1.917	1.845	1.780	1.725	1.678	1.635	1.596
1.000	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Table 2 - The Varied Flow Function Table (p.7 of 12)

u \ N	4.2	4.6	5.0	5.4	5.8	6.2	6.6	7.0	7.4	7.8
1.001	1.417	1.264	1.138	1.033	0.951	0.870	0.803	0.746	0.697	0.651
1.005	1.036	0.915	0.817	0.737	0.669	0.612	0.553	0.526	0.481	0.447
1.010	0.873	0.766	0.681	0.610	0.551	0.502	0.459	0.422	0.389	0.360
1.015	0.778	0.680	0.602	0.537	0.483	0.440	0.399	0.366	0.336	0.310
1.02	0.711	0.620	0.546	0.486	0.436	0.394	0.358	0.327	0.300	0.276
1.03	0.618	0.535	0.469	0.415	0.370	0.333	0.300	0.272	0.249	0.228
1.04	0.554	0.477	0.415	0.365	0.324	0.290	0.262	0.236	0.214	0.195
1.05	0.504	0.432	0.374	0.328	0.289	0.259	0.231	0.208	0.189	0.174
1.06	0.464	0.396	0.342	0.298	0.262	0.233	0.209	0.187	0.170	0.154
1.07	0.431	0.366	0.315	0.273	0.239	0.212	0.191	0.168	0.151	0.136
1.08	0.403	0.341	0.292	0.252	0.220	0.194	0.172	0.153	0.137	0.123
1.09	0.379	0.319	0.272	0.234	0.204	0.179	0.158	0.140	0.125	0.112
1.10	0.357	0.299	0.254	0.218	0.189	0.165	0.146	0.129	0.114	0.102
1.11	0.338	0.282	0.239	0.204	0.176	0.154	0.135	0.119	0.105	0.094
1.12	0.321	0.267	0.225	0.192	0.165	0.143	0.125	0.110	0.097	0.086
1.13	0.305	0.253	0.212	0.181	0.155	0.135	0.117	0.102	0.090	0.080
1.14	0.291	0.240	0.201	0.170	0.146	0.126	0.109	0.095	0.084	0.074
1.15	0.278	0.229	0.191	0.161	0.137	0.118	0.102	0.089	0.078	0.068
1.16	0.266	0.218	0.181	0.153	0.130	0.111	0.096	0.084	0.072	0.064
1.17	0.255	0.208	0.173	0.145	0.123	0.105	0.090	0.078	0.068	0.060
1.18	0.244	0.199	0.165	0.138	0.116	0.099	0.085	0.073	0.063	0.055
1.19	0.235	0.191	0.157	0.131	0.110	0.094	0.080	0.068	0.059	0.051
1.20	0.226	0.183	0.150	0.215	0.105	0.088	0.076	0.064	0.056	0.048
1.22	0.209	0.168	0.138	0.114	0.095	0.080	0.068	0.057	0.049	0.042
1.24	0.195	0.156	0.127	0.104	0.086	0.072	0.060	0.051	0.044	0.038
1.26	0.182	0.145	0.117	0.095	0.079	0.065	0.055	0.046	0.039	0.033
1.28	0.170	0.135	0.108	0.088	0.072	0.060	0.050	0.041	0.035	0.030
1.30	0.160	0.126	0.100	0.081	0.066	0.054	0.045	0.037	0.031	0.026
1.32	0.150	0.118	0.093	0.075	0.061	0.050	0.041	0.034	0.028	0.024
1.34	0.142	0.110	0.087	0.069	0.056	0.045	0.037	0.030	0.025	0.021
1.36	0.134	0.103	0.081	0.064	0.052	0.042	0.034	0.028	0.023	0.019
1.38	0.127	0.097	0.076	0.060	0.048	0.038	0.032	0.026	0.021	0.017
1.40	0.120	0.092	0.071	0.056	0.044	0.036	0.028	0.023	0.019	0.016
1.42	0.114	0.087	0.067	0.052	0.041	0.033	0.026	0.021	0.017	0.014
1.44	0.108	0.082	0.063	0.049	0.038	0.030	0.024	0.019	0.016	0.013
1.46	0.103	0.077	0.059	0.046	0.036	0.028	0.022	0.018	0.014	0.012
1.48	0.098	0.073	0.056	0.043	0.033	0.026	0.021	0.017	0.013	0.010
1.50	0.093	0.069	0.053	0.040	0.031	0.024	0.020	0.015	0.012	0.009
1.55	0.083	0.061	0.046	0.035	0.026	0.020	0.016	0.012	0.010	0.008
1.60	0.074	0.054	0.040	0.030	0.023	0.017	0.013	0.010	0.008	0.006

Table 2 - The Varied Flow Function Table (p.8 of 12)

u \ N	4.2	4.6	5.0	5.4	5.8	6.2	6.6	7.0	7.4	7.8
1.65	0.067	0.048	0.035	0.026	0.019	0.014	0.011	0.008	0.006	0.005
1.70	0.060	0.043	0.031	0.023	0.016	0.012	0.009	0.007	0.005	0.004
1.75	0.054	0.038	0.027	0.020	0.014	0.010	0.008	0.006	0.004	0.003
1.80	0.049	0.034	0.024	0.017	0.012	0.009	0.007	0.005	0.004	0.003
1.85	0.045	0.031	0.022	0.015	0.011	0.008	0.006	0.004	0.003	0.002
1.90	0.041	0.028	0.020	0.014	0.010	0.007	0.005	0.004	0.003	0.002
1.95	0.038	0.026	0.018	0.012	0.008	0.006	0.004	0.003	0.002	0.002
2.00	0.035	0.023	0.016	0.011	0.007	0.005	0.004	0.003	0.002	0.001
2.10	0.030	0.019	0.013	0.009	0.006	0.004	0.003	0.002	0.001	0.001
2.20	0.025	0.016	0.011	0.007	0.005	0.004	0.002	0.001	0.001	0.001
2.3	0.022	0.014	0.009	0.006	0.004	0.003	0.002	0.001	0.001	0.001
2.4	0.019	0.012	0.008	0.005	0.003	0.002	0.001	0.001	0.001	0.001
2.5	0.017	0.010	0.006	0.004	0.003	0.002	0.001	0.001	0.000	0.000
2.6	0.015	0.009	0.005	0.003	0.002	0.001	0.001	0.001	0.000	0.000
2.7	0.013	0.008	0.005	0.003	0.002	0.001	0.001	0.000	0.000	0.000
2.8	0.012	0.007	0.004	0.002	0.001	0.001	0.001	0.000	0.000	0.000
2.9	0.010	0.006	0.004	0.002	0.001	0.001	0.000	0.000	0.000	0.000
3.0	0.009	0.005	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000
3.5	0.006	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000
4.0	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.5	0.003	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.0	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.0	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7.0	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2 - The Varied Flow Function Table (p.9 of 12)

u \ N	8.2	8.6	9.0	9.4	9.8
0.00	0.000	0.000	0.000	0.000	0.000
0.02	0.020	0.020	0.020	0.020	0.020
0.04	0.040	0.040	0.040	0.040	0.040
0.06	0.060	0.060	0.060	0.060	0.060
0.08	0.080	0.080	0.080	0.080	0.080
0.10	0.100	0.100	0.100	0.100	0.100
0.12	0.120	0.120	0.120	0.120	0.120
0.14	0.140	0.140	0.140	0.140	0.140
0.16	0.160	0.160	0.160	0.160	0.160
0.18	0.180	0.180	0.180	0.180	0.180
0.20	0.200	0.200	0.200	0.200	0.200
0.22	0.220	0.220	0.220	0.220	0.220
0.24	0.240	0.240	0.240	0.240	0.240
0.26	0.260	0.260	0.260	0.260	0.260
0.28	0.280	0.280	0.280	0.280	0.280
0.30	0.300	0.300	0.300	0.300	0.300
0.32	0.320	0.320	0.320	0.320	0.320
0.34	0.340	0.340	0.340	0.340	0.340
0.36	0.360	0.360	0.360	0.360	0.360
0.38	0.380	0.380	0.380	0.380	0.380
0.40	0.400	0.400	0.400	0.400	0.400
0.42	0.420	0.420	0.420	0.420	0.420
0.44	0.440	0.440	0.440	0.440	0.440
0.46	0.460	0.460	0.460	0.460	0.460
0.48	0.480	0.480	0.480	0.480	0.480
0.50	0.500	0.500	0.500	0.500	0.500
0.52	0.520	0.520	0.520	0.520	0.520
0.54	0.540	0.540	0.540	0.540	0.540
0.56	0.561	0.560	0.560	0.560	0.560
0.58	0.581	0.581	0.580	0.580	0.580
0.60	0.601	0.601	0.601	0.600	0.600
0.61	0.611	0.611	0.611	0.611	0.610
0.62	0.621	0.621	0.621	0.621	0.621
0.63	0.632	0.631	0.631	0.631	0.631
0.64	0.642	0.641	0.641	0.641	0.641
0.65	0.652	0.652	0.651	0.651	0.651
0.66	0.662	0.662	0.662	0.661	0.661
0.67	0.673	0.672	0.672	0.672	0.671
0.68	0.683	0.683	0.682	0.682	0.681
0.69	0.694	0.693	0.692	0.692	0.692

Table 2 - The Varied Flow Function Table (p.10 or 12)

u \ N	8.2	8.6	9.0	9.4	9.8
0.70	0.704	0.704	0.703	0.702	0.702
0.71	0.715	0.714	0.713	0.713	0.712
0.72	0.726	0.725	0.724	0.723	0.723
0.73	0.736	0.735	0.734	0.734	0.733
0.74	0.747	0.746	0.745	0.744	0.744
0.75	0.758	0.757	0.756	0.755	0.754
0.76	0.769	0.768	0.767	0.766	0.765
0.77	0.780	0.779	0.778	0.777	0.776
0.78	0.792	0.790	0.789	0.788	0.787
0.79	0.804	0.802	0.800	0.799	0.798
0.80	0.815	0.813	0.811	0.810	0.809
0.81	0.827	0.825	0.823	0.822	0.820
0.82	0.839	0.837	0.835	0.833	0.831
0.83	0.852	0.849	0.847	0.845	0.844
0.84	0.865	0.862	0.860	0.858	0.856
0.85	0.878	0.875	0.873	0.870	0.868
0.86	0.892	0.889	0.886	0.883	0.881
0.87	0.907	0.903	0.900	0.897	0.894
0.88	0.921	0.918	0.914	0.911	0.908
0.89	0.937	0.933	0.929	0.925	0.922
0.90	0.954	0.949	0.944	0.940	0.937
0.91	0.972	0.967	0.961	0.957	0.953
0.92	0.991	0.986	0.980	0.975	0.970
0.93	1.012	1.006	0.999	0.994	0.989
0.94	1.036	1.029	1.022	1.016	1.010
0.950	1.062	1.055	1.047	1.040	1.033
0.960	1.097	1.085	1.074	1.063	1.053
0.970	1.136	1.124	1.112	1.100	1.087
0.975	1.157	1.147	1.134	1.122	1.108
0.980	1.187	1.175	1.160	1.150	1.132
0.985	1.224	1.210	1.196	1.183	1.165
0.990	1.275	1.260	1.243	1.228	1.208
0.995	1.363	1.342	1.320	1.302	1.280
0.999	1.560	1.530	1.500	1.476	1.447
1.000	∞	∞	∞	∞	∞

Table 2 - The Varied Flow Function Table (p.11 of 12)

u \ N	8.2	8.6	9.0	9.4	9.8
1.001	0.614	0.577	0.546	0.519	0.494
1.005	0.420	0.391	0.368	0.350	0.331
1.010	0.337	0.313	0.294	0.278	0.262
1.015	0.289	0.269	0.255	0.237	0.223
1.020	0.257	0.237	0.221	0.209	0.196
1.03	0.212	0.195	0.181	0.170	0.159
1.04	0.173	0.165	0.152	0.143	0.134
1.05	0.158	0.143	0.132	0.124	0.115
1.06	0.140	0.127	0.116	0.106	0.098
1.07	0.123	0.112	0.102	0.094	0.086
1.08	0.111	0.101	0.092	0.084	0.077
1.09	0.101	0.091	0.082	0.075	0.069
1.10	0.092	0.083	0.074	0.067	0.062
1.11	0.084	0.075	0.067	0.060	0.055
1.12	0.077	0.069	0.062	0.055	0.050
1.13	0.071	0.063	0.056	0.050	0.045
1.14	0.065	0.058	0.052	0.046	0.041
1.15	0.061	0.054	0.048	0.043	0.038
1.16	0.056	0.050	0.045	0.040	0.035
1.17	0.052	0.046	0.041	0.036	0.032
1.18	0.048	0.042	0.037	0.033	0.029
1.19	0.045	0.039	0.034	0.030	0.027
1.20	0.043	0.037	0.032	0.028	0.025
1.22	0.037	0.032	0.028	0.024	0.021
1.24	0.032	0.028	0.024	0.021	0.018
1.26	0.028	0.024	0.021	0.018	0.016
1.28	0.025	0.021	0.018	0.016	0.014
1.30	0.022	0.019	0.016	0.014	0.012
1.32	0.020	0.017	0.014	0.012	0.010
1.34	0.018	0.015	0.012	0.010	0.009
1.36	0.016	0.013	0.011	0.009	0.008
1.38	0.014	0.012	0.010	0.008	0.007
1.40	0.013	0.011	0.009	0.007	0.006
1.42	0.011	0.009	0.008	0.006	0.005
1.44	0.010	0.008	0.007	0.006	0.005
1.46	0.009	0.008	0.006	0.005	0.004
1.48	0.009	0.007	0.005	0.004	0.004
1.50	0.008	0.006	0.005	0.004	0.003
1.55	0.006	0.005	0.004	0.003	0.003
1.60	0.005	0.004	0.003	0.002	0.002

Table 2 - The Varied Flow Function Table (p.12 or 12)

u \ N	8.2	8.6	9.0	9.4	9.8
1.65	0.004	0.003	0.002	0.002	0.001
1.70	0.003	0.002	0.002	0.001	0.001
1.75	0.002	0.002	0.002	0.001	0.001
1.80	0.002	0.001	0.001	0.001	0.001
1.85	0.002	0.001	0.001	0.001	0.001
1.90	0.001	0.001	0.001	0.001	0.000
1.95	0.001	0.001	0.001	0.000	0.000
2.00	0.001	0.001	0.000	0.000	0.000
2.10	0.001	0.000	0.000	0.000	0.000
2.20	0.000	0.000	0.000	0.000	0.000
2.3	0.000	0.000	0.000	0.000	0.000
2.4	0.000	0.000	0.000	0.000	0.000
2.5	0.000	0.000	0.000	0.000	0.000
2.6	0.000	0.000	0.000	0.000	0.000
2.7	0.000	0.000	0.000	0.000	0.000
2.8	0.000	0.000	0.000	0.000	0.000
2.9	0.000	0.000	0.000	0.000	0.000
3.0	0.000	0.000	0.000	0.000	0.000
3.5	0.000	0.000	0.000	0.000	0.000
4.0	0.000	0.000	0.000	0.000	0.000
4.5	0.000	0.000	0.000	0.000	0.000
5.0	0.000	0.000	0.000	0.000	0.000
6.0	0.000	0.000	0.000	0.000	0.000
7.0	0.000	0.000	0.000	0.000	0.000
8.0	0.000	0.000	0.000	0.000	0.000
9.0	0.000	0.000	0.000	0.000	0.000
10.0	0.000	0.000	0.000	0.000	0.000
20.0	0.000	0.000	0.000	0.000	0.000

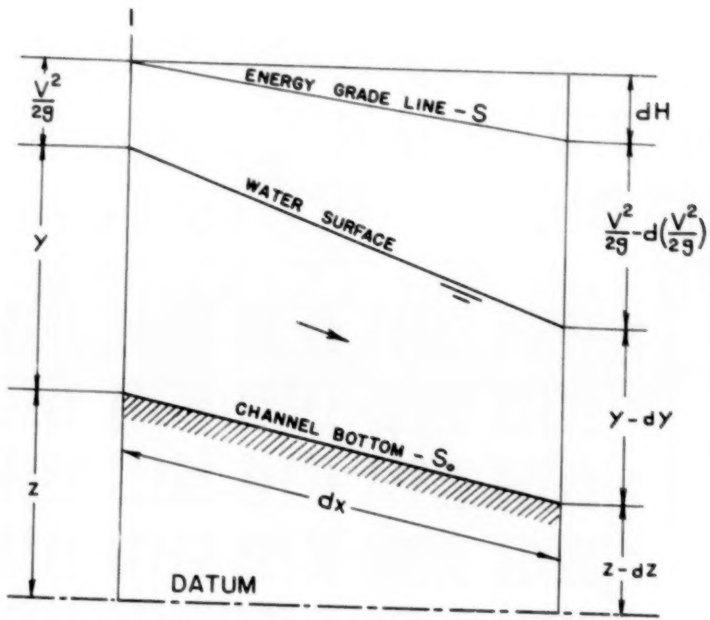

FIG. I GRADUALLY VARIED FLOW IN A VERY SHORT REACH dx

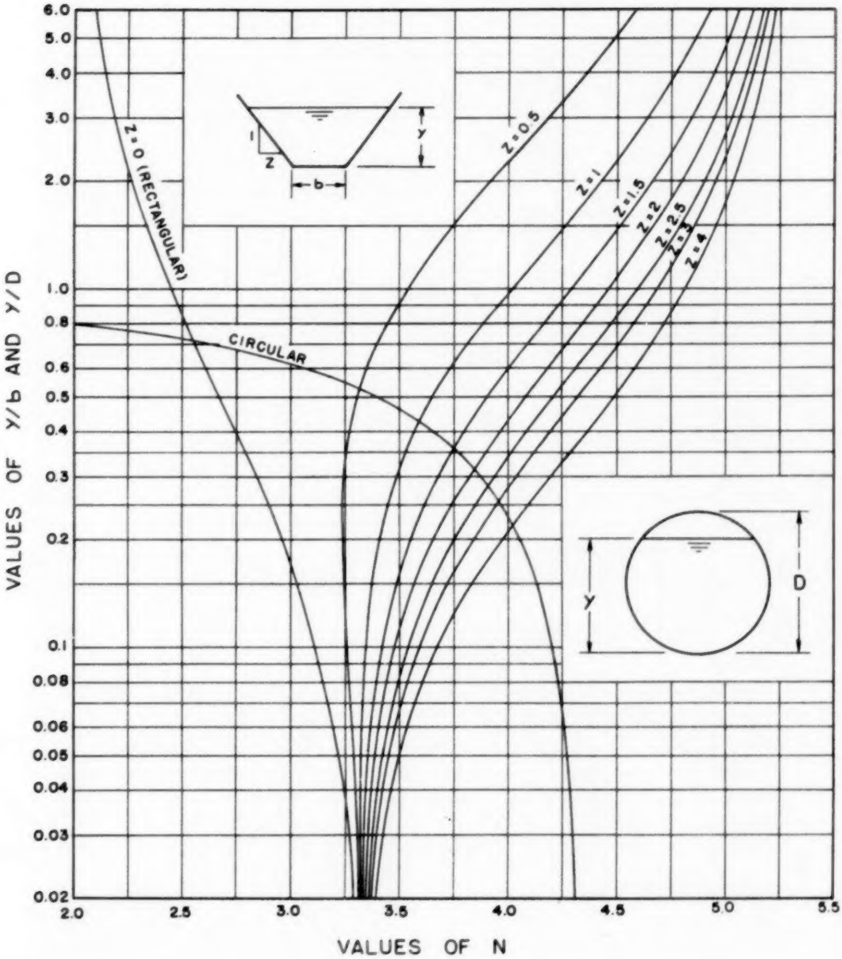

Fig. 2. Curves of N-Values

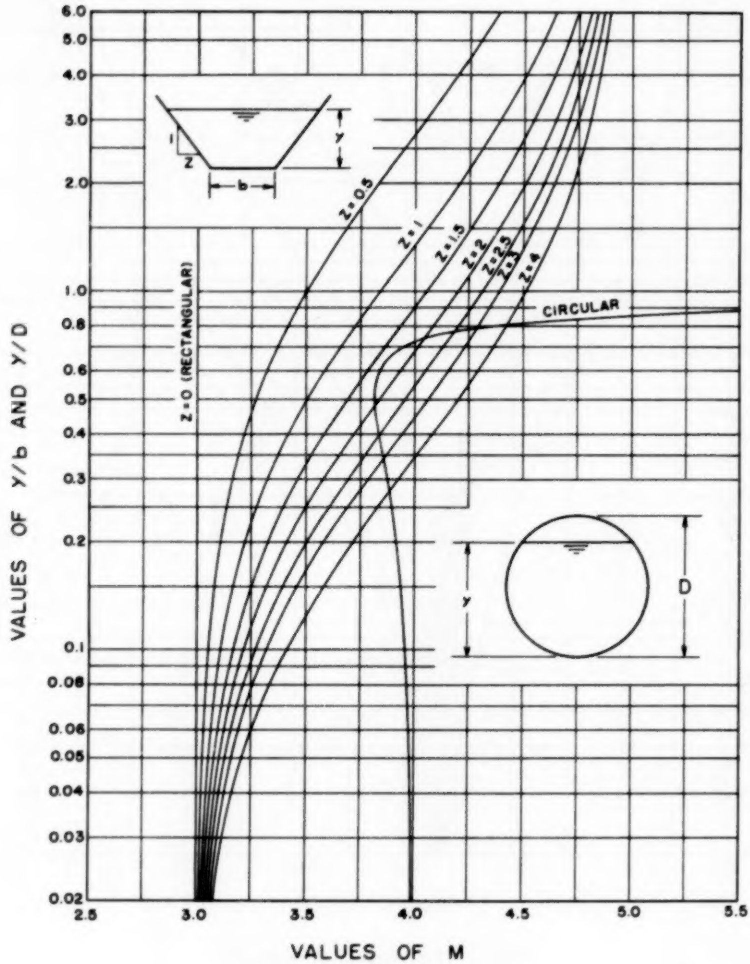

Fig. 3. Curves of M-Values

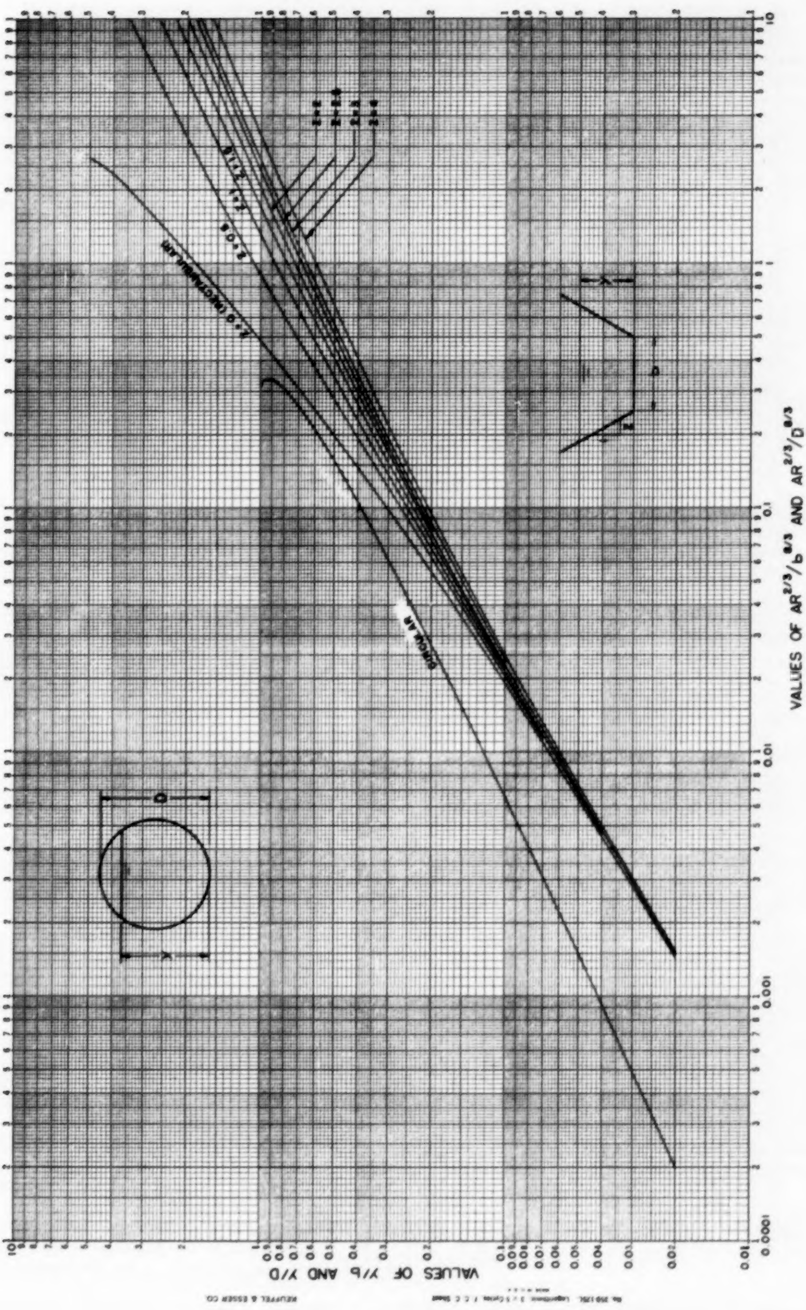

Fig. 4. Curves for Determining the Normal Depth

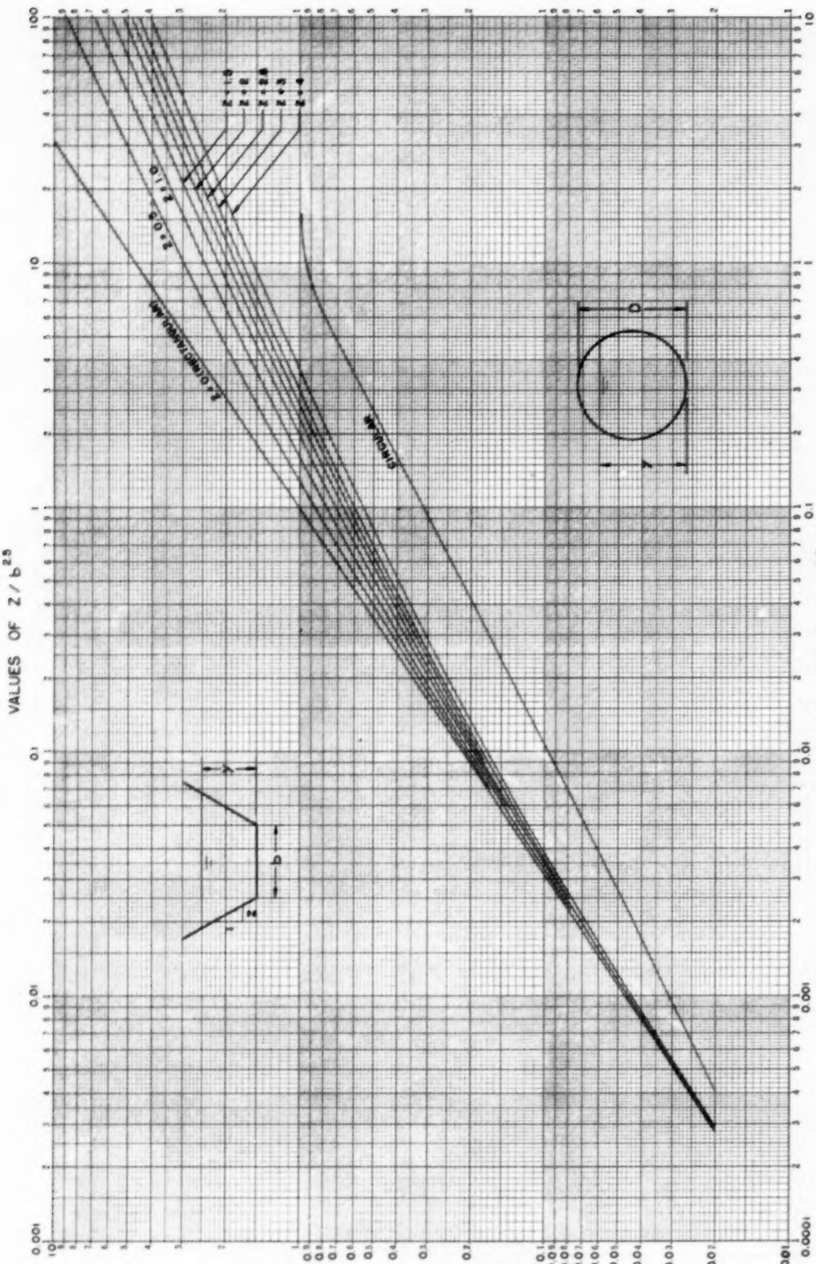

Fig. 5. Curves for Determining the Critical Depth

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. Papers sponsored by the Board of Direction are identified by the symbols (BD). For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(BD).

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JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)[c], 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)[c], 596(HW), 597(HW), 598(HW)[c], 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)[c], 607(EM).

FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)[c], 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)[c], 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)[c], 655(SA), 656(SM)[c], 657(SM)[c], 658(SM)[c].

APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

MAY: 679(ST), 680(ST), 681(ST), 682(ST)[c], 683(ST), 684(ST), 685(SA), 686(SA), 687(SA), 688(SA), 689(SA)[c], 690(EM), 691(EM), 692(EM), 693(EM), 694(EM), 695(EM), 696(PO), 697(PO), 698(SA), 699(PO)[c], 700(PO), 701(ST)[c].

JUNE: 702(HW), 703(HW), 704(HW)[c], 705(IR), 706(IR), 707(IR), 708(IR), 709(HY)[c], 710(CP), 711(CP), 712(CP), 713(CP)[c], 714(HY), 715(HY), 716(HY), 717(HY), 718(SM)[c], 719(HY)[c], 720(AT), 721(AT), 722(SU), 723(WW), 724(WW), 725(WW), 726(WW)[c], 727(WW), 728(IR), 729(IR), 730(SU)[c], 731(SU).

JULY: 732(ST), 733(ST), 734(ST), 735(ST), 736(ST), 737(PO), 738(PO), 739(PO), 740(PO), 741(PO), 742(PO), 743(HY), 744(HY), 745(HY), 746(HY), 747(HY), 748(HY)[c], 749(SA), 750(SA), 751(SA), 752(SA)[c], 753(SM), 754(SM), 755(SM), 756(SM), 757(SM), 758(CO)[c], 759(SM)[c], 760(WW)[c].

AUGUST: 761(BD), 762(ST), 763(ST), 764(ST), 765(ST)[c], 766(CP), 767(CP), 768(CP), 769(CP), 770(CP), 771(EM), 772(EM), 773(SA), 774(EM), 775(EM), 776(EM)[c], 777(AT), 778(AT), 779(SA), 780(SA), 781(SA), 782(SA)[c], 783(HW), 784(HW), 785(CP), 786(ST).

SEPTEMBER: 787(PO), 788(IR), 789(HY), 790(HY), 791(HY), 792(HY), 793(HY), 794(HY)[c], 795(EM), 796(EM), 797(EM), 798(EM), 799(EM)[c], 800(WW), 801(WW), 802(WW), 803(WW), 804(WW), 805(WW), 806(HY), 807(PO)[c], 808(IR)[c].

OCTOBER: 809(ST), 810(HW)[c], 811(ST), 812(ST)[c], 813(ST)[c], 814(EM), 815(EM), 816(EM), 817(EM), 818(EM), 819(EM)[c], 820(SA), 821(SA), 822(SA)[c], 823(HW), 824(HW).

NOVEMBER: 825(ST), 826(HY), 827(ST), 828(ST), 829(ST), 830(ST), 831(ST)[c], 832(CP), 833(CP), 834(CP), 835(CP)[c], 836(HY), 837(HY), 838(HY), 839(HY), 840(HY), 841(HY)[c].

c. Discussion of several papers, grouped by Divisions.

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